which includes the longitudinal reference length C_0, as expected. The physical meanings of Eq. (34) might be inferred in the following way: if the angle-of-attack is positive, a forward acceleration of a wing surface brings the downward component normal to the wing surface. Then, according to the theory of out-of-plane wing motions, a lift due to virtual mass effect is produced. It is easy to show that this reasoning is valid for a flat delta wing. In fact, from Δp_l we have the following part

$$\Delta p_l = 2\rho(-f_{Ltt})\sqrt{b^2-y^2} \tag{36}$$

which is equivalent to Eq. (31) if we replace $\alpha\dot{U} = \alpha U\tilde{U}'$ by $-f_{Ltt}$.

IV. Conclusions

The lift force of a slender wing is analytically determined for nonuniform flight speed. The fluid is assumed to be inviscid. The flight speed may be either subsonic, transonic, or supersonic. It is found that the lift depends on the instantaneous acceleration and not on the history of the wing motion. An acceleration (or deceleration) increases (decreases) the lift force from that of the steady-state. The reason for this may be attributed to the component of a forward acceleration of a wing normal to the surface.

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High-Frequency Subsonic Flow Past a Pulsating Thin Airfoil II: Gust-Type Upwash

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Introduction

In Ref. 1, the solution was given for the high-frequency subsonic potential flow past a thin airfoil pulsating harmonically in time. This closed-form high-frequency approximation is an example of a symmetric (nonlifting) disturbance to a subsonic stream and, as such, complements the existing solutions for lifting problems (see, for example, Ref. 2). The possible applications are in aeroacoustics. In this Note, the results are extended to treat an example where the scale of the streamwise variation of the airfoil shape is comparable to the acoustic wavelength; the interesting case of a Sears gust-type upwash is studied.

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Analysis

The first-order perturbation velocity potential for two-dimensional flow of a uniform stream of speed U, Mach number $M(\beta^2 \equiv 1-M^2)$, and sound speed a past a pulsating airfoil with surface

$$y = \pm\epsilon g(x)\exp(i\omega t); \quad -\ell \le x \le \ell \tag{1}$$

is given in Ref. 1 as

$$\epsilon\phi^{(1)}(x,y)\exp(i\omega t) = i\epsilon(2\beta)^{-1}\int_{-\ell}^{\ell}[i\omega g(\xi)+Ug'(\xi)]$$
$$\times\exp\left[\frac{i\omega M(x-\xi)}{a\beta^2}\right]H_0^{(2)}\left\{\frac{\omega[(x-\xi)^2+\beta^2y^2]^{1/2}}{a\beta^2}\right\}d\xi$$
$$\times\exp(i\omega t) \tag{2}$$

In Ref. 1, the asymptotic expansion of Eq. (2) was given for $\omega\to\infty$ with $g(x)$ and $g'(x)$ of $O(1)$. Consider pulsation where $g(x)$ varies on the scale of the acoustic wavelength $\lambda = 2\pi a\omega^{-1}$. It is of theoretical interest to choose the upwash to be the symmetrical counterpart of that for the Sears-type compressible gust:

$$\phi_y^{(1)}(x,0\pm) = \pm w_0 e^{-i\omega x/U}; \quad -\ell \le x \le \ell \tag{3}$$

This upwash is realized if the airfoil shape function is chosen to be

$$g(x) = w_0U^{-1}xe^{-i\omega x/U} \tag{4}$$

Substitution into Eqs. (2) and (4) yields

$$\phi^{(1)} = iw_0(2\beta)^{-1}\exp\left(-\frac{i\omega x}{U}\right)\int_{-\ell}^{\ell}\exp\left[\frac{i\omega(x-\xi)}{Ma\beta^2}\right]$$
$$\times H_0^{(2)}\left\{\frac{\omega[(x-\xi)^2+\beta^2y^2]^{1/2}}{a\beta^2}\right\}d\xi \tag{5}$$

There is a contribution to the asymptotic value of the integral from an interior critical point and the two end points. The following results will be invalid for $|x+\ell| = O(\lambda)$ and $|x-\ell| = O(\lambda)$ due to the coalescing of the interior critical point with one of the end points. An appropriate scale for y is λ, and therefore a solution is sought in the outer region $y = O(\lambda)$. In the evaluation of the integral in Eq. (5), the variable $Y = \omega y$, which is $O(1)$, is used.

The solution is written as

$$\phi^{(1)} = \phi^{(1I)} + \phi^{(1L)} + \phi^{(1T)} \tag{6}$$

where the superscripts I, L, and T represent the interior critical point, forward end point, and rear end point. The interior critical point is a zero at $x = \xi$. Its contribution is obtained by using Eq. (6.4.9) in Ref. 3 to yield

$$\phi^{(1I)} = \begin{cases} -Uw_0\omega^{-1}\exp[-\omega(ix+|y|)U^{-1}]; & |x|<\ell \\ 0; & |x|>\ell \end{cases} \tag{7}$$

To evaluate the end point contributions, the asymptotic expansion for large argument of the Hankel function is introduced into Eq. (5). After some manipulation, the results for the forward and rear end points are evaluated with the help of Eqs. (6.3.28) and (6.3.38) in Ref. 3 to yield

$$\phi^{(IL)} = iUw_0\omega^{-3/2}(a/2\pi)^{1/2}(1 \pm M)[(x+\ell)^2+\beta^2y^2]^{-1/4}$$
$$\times \exp\left\{\frac{-i\omega y^2}{2a|x+\ell|} \mp \frac{i\omega x}{a(1\pm M)} + \frac{i\omega\ell}{U(1\pm M)} - \frac{i\pi}{4}\right\} \qquad (8)$$

where the upper sign is for $x > -\ell$ and the lower for $x < -\ell$, and

$$\Phi^{(IT)} = -iUw_0\omega^{-3/2}(a/2\pi)^{1/2}(1 \pm M)[(x-\ell)^2+\beta^2y^2]^{-1/4}$$
$$\times \exp\left\{\frac{-i\omega y^2}{2a|x-\ell|} \mp \frac{i\omega x}{a(1\pm M)} - \frac{i\omega\ell}{U(1\pm M)} - \frac{i\pi}{4}\right\} \qquad (9)$$

where the upper sign is for $x > \ell$ and the lower for $x < \ell$.

In the immediate neighborhood of the airfoil, $\phi^{(1)}$ in Eq. (6) can be obtained by setting $y=0$ in Eqs. (7-9). For high frequency, the pressure on the airfoil surface is given by

$$p = -\rho\epsilon(i\omega\phi^{(1)} + U\phi_x^{(1)})\exp(i\omega t) \qquad (10)$$

where the appropriate solutions are used for $|x| < \ell$, and ρ is the density in the stream. Substitution of Eqs. (8 and 9) yields

$$p = \rho\epsilon Uw_0\omega^{-1/2}(a/2\pi)^{1/2}\exp(-i\pi/4)$$
$$\times\left\{(x+\ell)^{-1/2}\exp\left[\frac{i\omega\ell}{U(1+M)}\right]\exp\left[i\omega\left(t-\frac{x}{a(1+M)}\right)\right]\right.$$
$$\left.-(x-\ell)^{-1/2}\exp\left[\frac{-i\omega\ell}{U(1-M)}\right]\exp\left[i\omega\left(t+\frac{x}{a(1-M)}\right)\right]\right\} \qquad (11)$$

Discussion

The first-order unsteady velocity potential for the high-frequency subsonic flow past a pulsating airfoil with a gust-type upwash is given in Eqs. (6-9). It is valid in a region whose extent in the transverse direction is of the order of the wavelength. The solution has a different representation for the regions upstream of the leading edge, over the airfoil, and downstream of the trailing edge. The contributions from all of the critical points are wavelike in character and can be described best near to the foil. For $y=0$, the solution in Eq. (7) represents a wave traveling downstream with the freestream speed and an amplitude that is nonzero only above the airfoil. The solution in Eq. (8) represents upstream and downstream waves radiating from the leading edge with speeds $a(1-M)$ and $a(1+M)$, respectively. The solution in Eq. (9) similarly represents upstream and downstream waves radiating from the trailing edge with the same speeds.

The flowfield has the following unsteady components. Upstream of the leading edge, the solution consists of the upstream waves from the airfoil edges. Downstream of the trailing edge, the solution consists of the downstream waves from the airfoil edges. Over the airfoil, the solution consists of the downstream wave with speed U, the downstream wave from the leading edge, and the upstream wave from the trailing edge. The solutions are invalid in the neighborhood of the airfoil edges, and the asymptotic evaluation of the integrals in this region is beyond the scope of this research.

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Carbon Vaporization into a Nonequilibrium, Stagnation-Point Boundary Layer

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Nomenclature

C	$=\rho\mu/(\rho\mu)_e$, ratio of density-viscosity product
c_p	= specific heat at constant pressure of the mixture
$\bar{c}_p$	$= c_p/c_{p,e}$
$c_{p,i}$	= specific heat at constant pressure of ith species
$\bar{c}_{p,i}$	$= c_{p,i}/c_{p,e}$
D_ℓ	= dissociation energy of ℓ th step
f	= Blasius function (dimensionless stream function)
h_i	= specific enthalpy of ith species
K	= number of elementary reaction steps
$K_{b,\ell}$	= backward reaction of ℓ th step
$K_{f,\ell}$	= forward reaction rate of ℓ th step
$K_{p,\ell}$	= equilibrium constant of ℓ th step
M	= mean molecular weight of the mixture
M_i	= molecular weight of ith species
$M_{\ell i}$	= reduced mass
$\dot{m}_i$	= vaporization rate of ith species
$\bar{\dot{m}}_i$	$=(\dot{m}_i/\sqrt{2\rho_e\mu_e\alpha})$, dimensionless vaporization rate
N	= number of species
N_A	= Avogadro's number
Pr	= Prandtl number
$P_{v,i}$	= equilibrium vapor pressure of ith species
R	= universal gas constant
Sc	= Schmidt number
S_i	= specific entropy of ith species
S_∞	= collision cross section
$S_{\ell j}$	= symmetric number ($S_{\ell j}=1$ for $\ell \neq j$, and $S_{\ell j}=2$ for $\ell = j$)
T	= temperature
U_e	= freestream velocity
W_i	= chemical production rate
x	= coordinate parallel to the surface
X_i	= ith chemical species
y	= coordinate normal to the surface
Y_i	= mass fraction of ith species
α	$=(dU_e/dx)_{x=0}$
η	$=\left(\frac{2\rho_e\alpha}{\mu_e}\right)^{1/2}\int_0^y \frac{\rho}{\rho_e}dy$
ρ	= density of the mixture

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